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NEW METHODS FOR NONLINEAR TRACKING AND NONLINEAR CHAOTIC SIGNAL PROCESSING

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The aim of the current research is to establish that we can identify/classify the individual signals that make up the total signal. The algorithm can function either as a detection algorithm or as an algorithm for preliminary processing leading to actual separation and cleaning of signals. It is not currently possible to do this latter task, even in the simplest situation, without first determining what signals are to be separated. We imagine that we have a library of samples of signals from sources of interest. An incoming signal is received that is a sum of one or more of the signals in the library plus noise, and we wish to determine which signals are actually present (i.e., we want to identify the source of the signal) and what are the relative amplitudes of the signals. We assume that some, perhaps all, of the signals have a broad frequency spectrum so that conventional linear filtering is inadequate. It is further assumed that we have samples of the individual signals in a library, although if they are broad-band chaotic, the signals will not be identical. One or more of the signals may be noise with its own characteristics. To illustrate that we can actually accomplish this, we will present the results of some simulations. In the figures on the following pages are some samples of the signals we have been working with. Figures 1 and 2 present time traces and Figures 3 and 4 are the corresponding spectra. The signals labelled 3 and 7 are random gaussian noise with a spectrum chosen to coincide with that of one of the other signals. Two of the signals, 4 and 5, are from electronic circuits and the rest are the solutions of some nonlinear differential equations. From the spectra we can see that there is little possibility of separation by filtering.

In order to test our method, we form combinations of two signals to give a third,

$$S(t) = a\,P(t) + b\,Q(t)$$

The coefficients a and b are to be determined as well as which signals P and Q correspond to. Since we measure S(t) we know its level so that we know the value of the sum of the squares of a and b. For the particular example that we show here, a = .48 and b = .87 .

We have been using a chi-squared statistics as a figure of merit, and the appropriate function of the densities of the signals is compared. In this case we constructed a two-dimensional density by using a signal and a time-lagged signal for the two components.

The criterion that we use is exact in the limit of an infinitely long signal, but in the present case a relatively short signal was used. This means that even when the correct signals with the correct amplitudes are compared there will be some residual statistical error. In the lower right-hand graph of Figure 5, chi-squared is shown versus the relative strength parameter, α. There are two curves that are nearly identical and one that has a larger chi-squared. One of the two lower curves correspond to the exact comparison and give us an estimate of the statistical uncertainty that places a lower limit on chi-squared. The second curve was computed by taking a different sample of signals 4 and 6 and comparing various combinations of them with the densities of the original signal We see that the value of α is clearly about .5, which is

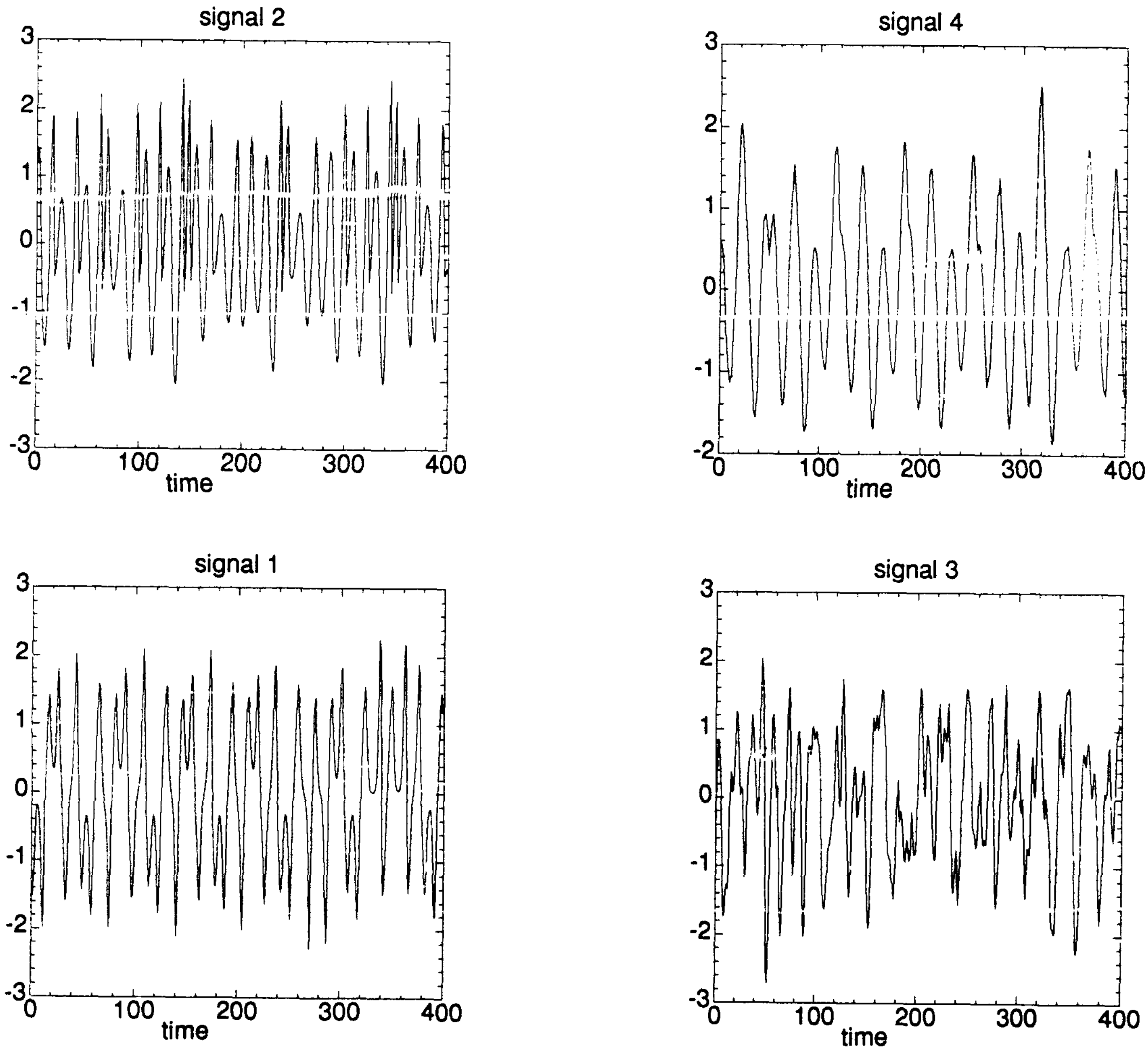

Fig. 1. Time traces of four of the signals used in this demonstration. Signal 3 is random gaussian noise with the same spectrum as one of the other signals.

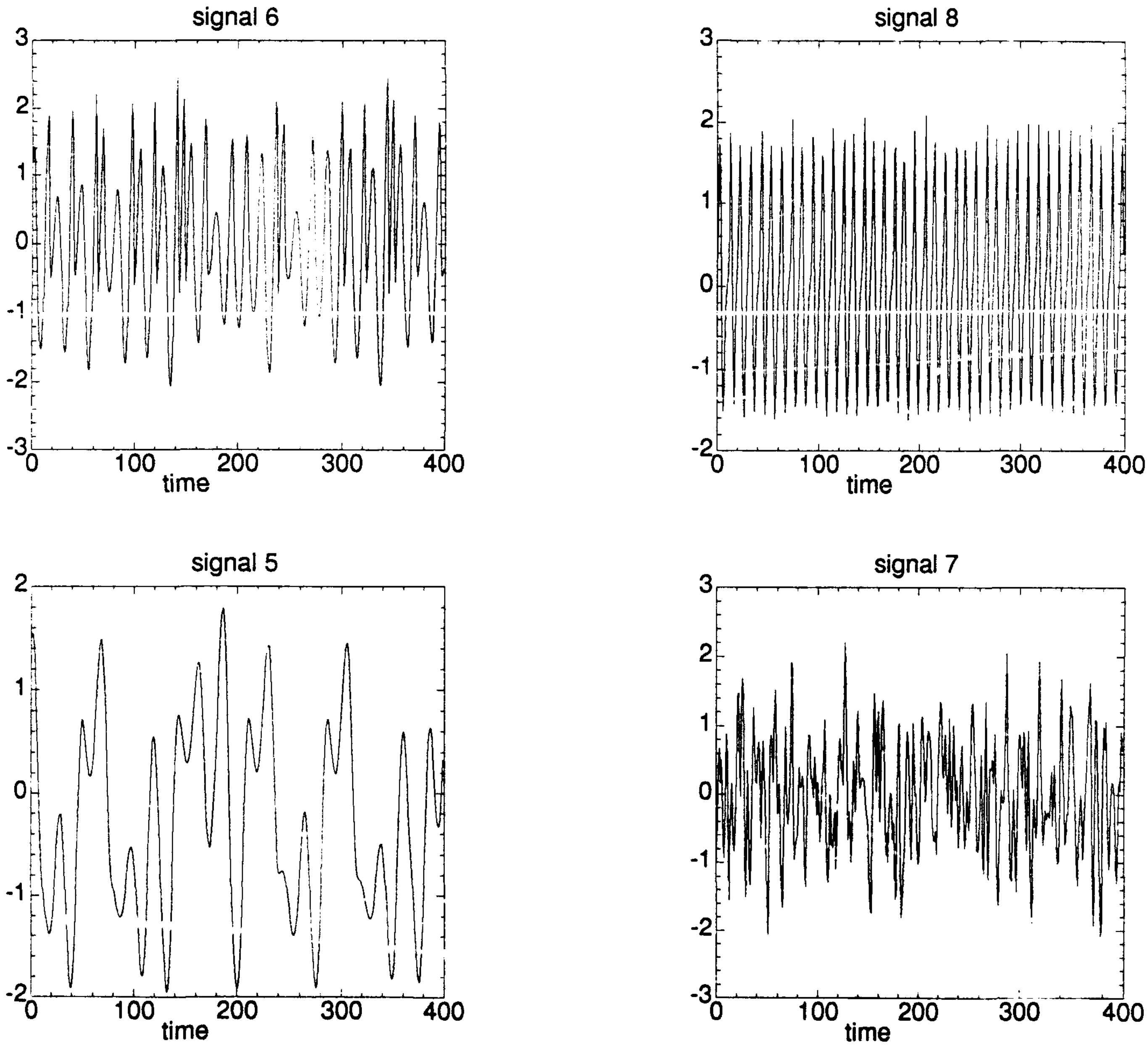

Fig. 2. Time traces of four of the signals used in this demonstration. Signal 7 is random gaussian noise with the same spectrum as one of the other signals.

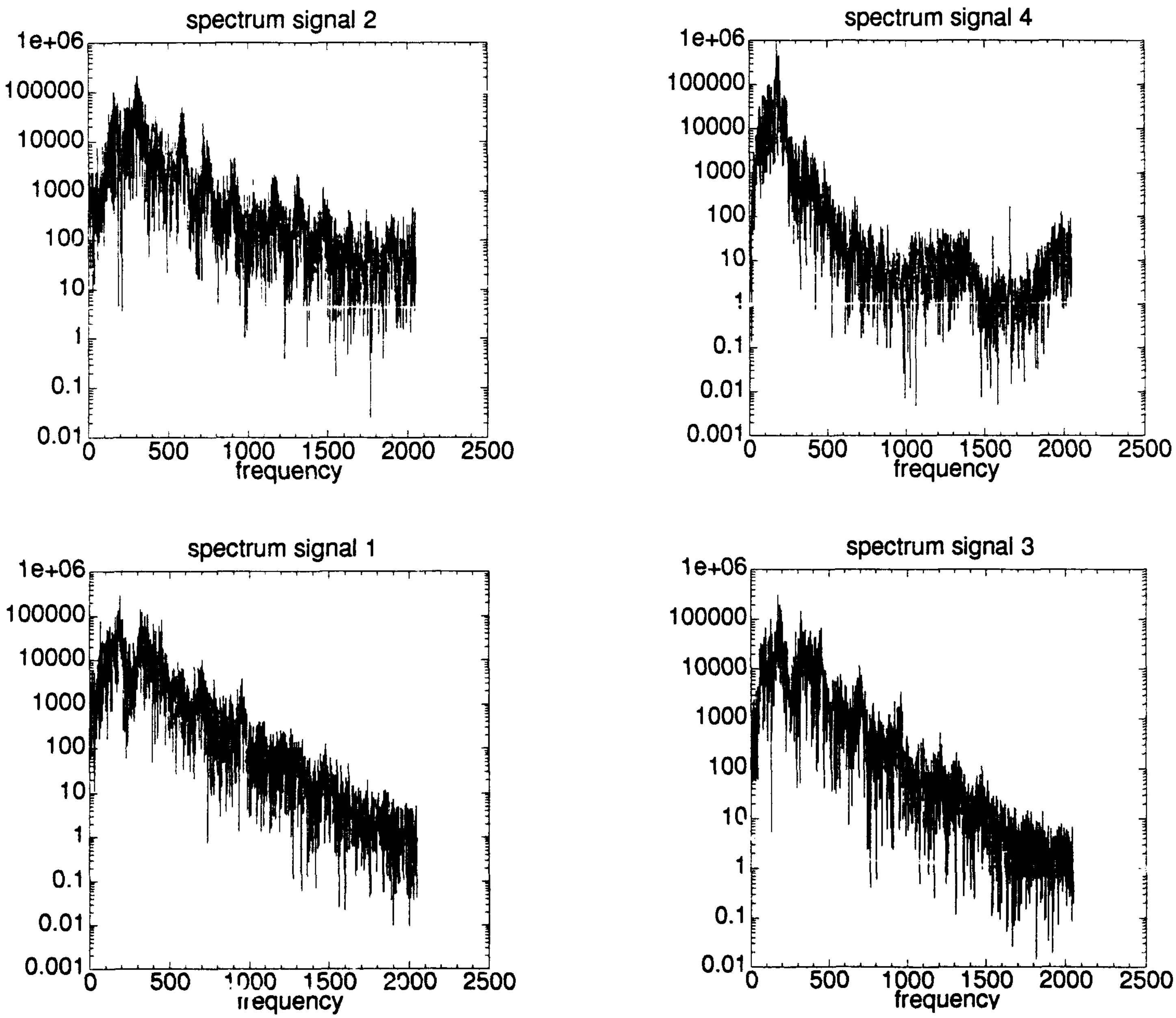

Fig. 3. The corresponding power spectra for the signals shown in Figure 1.

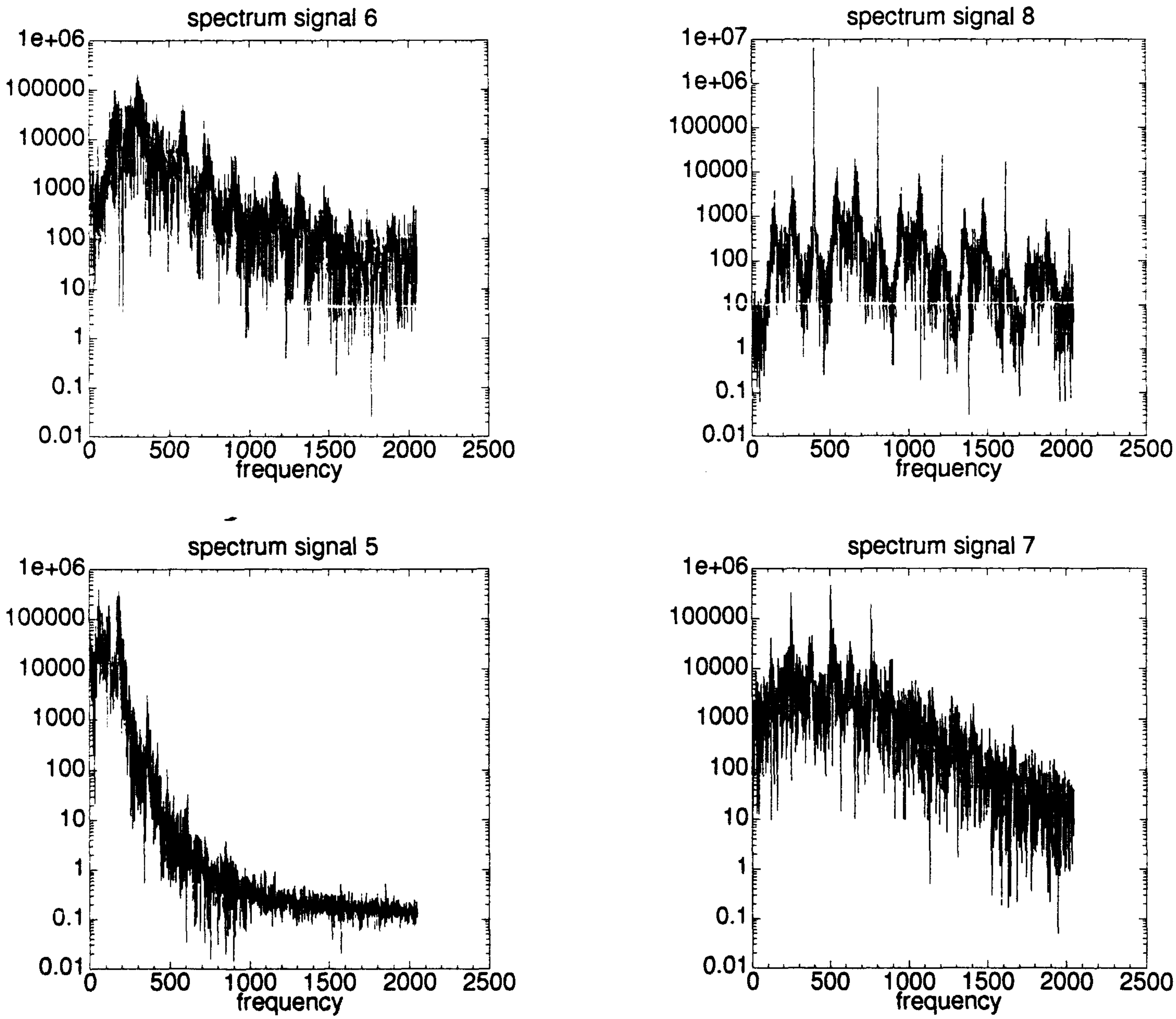

Fig. 4. The corresponding power spectra for the signals shown in Figure 2.

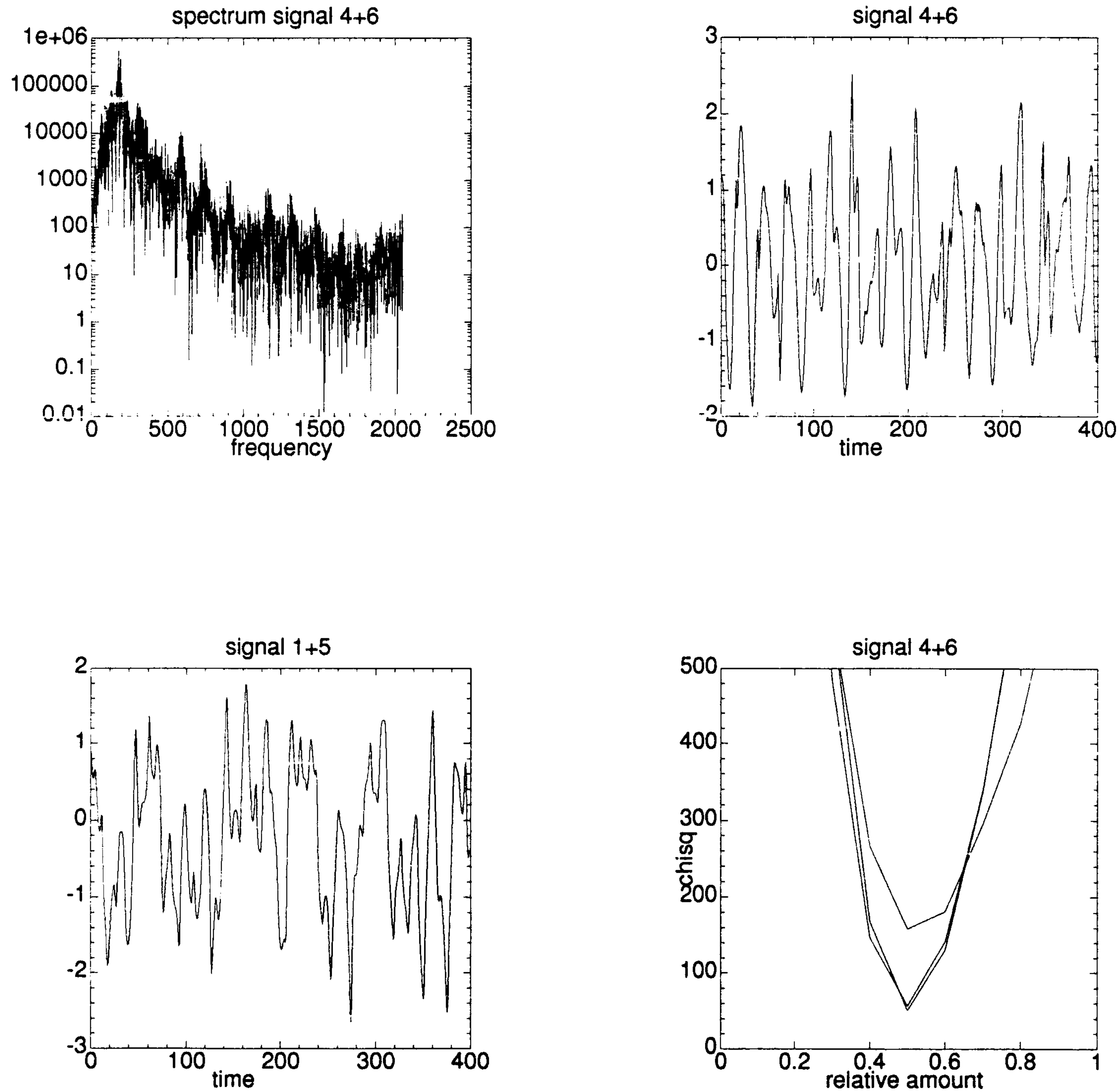

Fig. 5. (a) is the power spectrum of the signal shown in 5b; (b) is the sum of signals 4 + 6 with the relative strength parameters a and b given in the text; (c) Different pair of signals that was being compared with signals 4 + 6; (d) Chi-squared is a figure of merit for how good the fit is, smaller chi-squared being better. The horizontal axis is a relative amount of the two signals being combined. The two curves that nearly touch are two samples of signals 4 + 6. The upper curve is for signal 6 + another unshown signal, and the chi-squared for signals 1 + 5 would be off the graph.

correct. The residual error is due to the inherent lack of statistics. The upper curve corresponds to the hypothesis that one of the signals was correct and the other incorrect. Note that even in this case our algorithm correctly identifies the amount of the right signal. We then considered two different signals, neither of which corresponded to the original choice, namely signals 1 plus 5 (see lower left of Figure 5). In this case chi-squared was relatively flat and well above the levels shown on the graph, thus affirming that neither signal was present in the signal to be identified.

The amount of data used is of course important. The computational effort is proportional to the length of the time the signal is observed, and there is the logistic problem of not being able to collect an infinite amount of data in a finite time. For the examples shown, we sampled about ten times per oscillation and included a total of 10,000 points. For typical machinery oscillations of 1000 rpm, this is about a minute's worth of data.

Depending upon the types of signals of interest, one may get by with less total samples. We tried many different combinations of signals, even including noise in the signal to be identified, and all of our results were comparable to the example presented above. We were even able to recognize that a signal was made up of two similar signals so that if there are two sources of the same signal we could recognize that fact.

It is important to be able to identify a weak signal combined with a strong signal. The limitations on this are not known at the present and will undoubtedly be dependent upon refinements of the algorithm. Qualitatively we have no problem separating signals with a ratio of energies of ten to one. Usually we can resolve power levels of a hundred to one, and we expect further improvement will be possible.

We have done some limited testing with a signal made up of three signals. In the first example one signal was broadband noise (signal 7) and then two other chaotic signals. The energy in each signal was the same. The correct combination was easily selected out. We then tried a combination with the energies in the ratio of 9 to 4 to 1, with the 9 being the broad-band noise. There did not seem to be any problem identifying the two signals even in the presence of noise. There was one example of an alternative signal giving as good a statistic as one of the correct signals. In order to resolve this discrepancy we performed a separate test which completely resolved the uncertainty. It is encouraging that almost all incorrect combinations were easily rejected in the two-dimensional phase space, as this means that only a few candidates might have to be further processed as we did in the one example in order to get better rejection of incorrect hypotheses.